

# Verification of Validity of Logical Syllogisms with New Forms of Intermediate Quantifiers Based on Grades

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## Abstract

In this contribution, we continue our investigation of fuzzy Peterson syllogisms. Whereas the previous study concentrated on validating these syllogisms through the construction of formal proofs and semantic verification, the present work focuses on assessing their validity using Peterson's grade-based rules.

**Keywords:** Fuzzy Peterson's syllogisms, Intermediate quantifiers, Graded Peterson's square of opposition

## 1 Introduction

In our previous two publications, we addressed the generalized Peterson rules. First, we formally introduced their mathematical definitions based on the logical operations of Łukasiewicz algebra [1], and subsequently applied these rules to verify the validity and invalidity of logical syllogisms [2]. In the present contribution, we build upon earlier results [3], where we proposed a more general mathematical formulation of linguistically defined Peterson rules. In contrast to the previous study [1], the proposed rules do not require an explicit mathematical definition of intermediate quantifiers; instead, they rely solely on the relative position of a given quantifier within the graded Peterson square of opposition (see [4]). The novelty of this contribution is the application of other forms of intermediate quantifiers and the application of an algorithm for verifying the validity or invalidity of logical syllogisms.

## 2 Mathematical background

Since we are limited by the number of pages, we will only provide the most necessary mathematical definitions in order to be able to present concrete results. Therefore, we refer the reader to previous publications.

We will work with the standard Łukasiewicz-algebra.

$$\mathcal{L} = \langle [0, 1], \vee, \wedge, \otimes, \rightarrow, 0, 1 \rangle. \quad (1)$$

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## 2.1 Extended Peterson's rules

Below we recall the mathematical definitions of the rules of distributivity, quality and quantity <sup>†</sup>. We need to know if the quantifier is *affirmative or negative*. Furthermore, we need to know a *position* of quantifier. It means to know the position inside of the graded Peterson's square [5].

A set of all considered quantifiers will be denoted by  $\mathcal{Q}$ . For Peterson's framework of generalized intermediate syllogisms, we have

$$\mathcal{Q} = \{\text{"all", "almost all", "most", "many", "A few", "Several", "some"}.\}$$

**Definition 1 (Proposition)** Proposition is an expression in the form either

$$Q(B, A) \quad \text{or} \quad Q(B, \neg A)$$

where  $Q$  is a  $\langle 1, 1 \rangle$  quantifier and  $A, B$  are formulas.  $Q(B, A)$  is an affirmative and  $Q(B, \neg A)$  is a negative proposition.

Each quantifier  $Q \in \mathcal{Q}$  must have assigned a *quantity*( $Q$ ) which is explained in the subsequent definition. By *quantity*( $Q^{\text{almost all}}$ ) we, for example, denote the quantity of the quantifier "Almost all".

**Definition 2 (Quantity)** Let  $Q$  be a quantifier. We say that *quantity*( $Q$ ) fulfills the properties as follows:

- (a)  $0 < \text{quantity}(Q) \leq 1$  for all  $Q \in \mathcal{Q}$ ;
- (b)  $\text{quantity}(Q_1) \leq \text{quantity}(Q_2)$  iff for  $Q_1, Q_2 \in \mathcal{Q}$ , proposition  $Q_2(B, A)$  is superaltern of proposition  $Q_1(B, A)$ , i.e.,  $\mathcal{M}(Q_2(B, A)) \leq \mathcal{M}(Q_1(B, A))$ ;
- (c)  $\text{quantity}(Q) > 0.5$  iff for  $Q \in \mathcal{Q}$ , proposition  $Q(B, A)$  is contrary to  $Q(B, \neg A)$ ;
- (d)  $\text{quantity}(Q_1) + \text{quantity}(Q_2) > 1$  if  $Q_1(B, A)$  and  $Q_2(B, \neg A)$  form a contradictory pair.

### 2.1.1 Grade

In Peterson's approach, the distribution index is based on the number of intermediate quantifiers. It means that the maximal value is 5 and the other values depend on the position in the Peterson's square of opposition. Our approach is based on the size that the quantifier represents in the Peterson square. In our approach, we represent this size by a **grade**, which is a value from an interval  $[0, 1]$ .

**Definition 3** Let us assume five basic Peterson's quantifiers. Then *quantity of quantifier* is represented by *grade* as follows:

- $\text{quantity}(Q^{\text{All}}) = 1$ ,  $\text{quantity}(Q^{\text{Almost all}}) = p$ ,  $\text{quantity}(Q^{\text{Most}}) = t$ ,
- $\text{quantity}(Q^{\text{Many}}) = k$ ,  $\text{quantity}(Q^{\text{A few}}) = f$ ,  $\text{quantity}(Q^{\text{Several}}) = s$ ,
- $\text{quantity}(Q^{\text{Some}}) = \epsilon$

such that

<sup>†</sup>Recall that all these properties were discussed in great detail in a previous publication (see [4]).

(a)  $0 < \epsilon < s < f < k < 0.5 < t < p < 1 - \epsilon < 1$ ,  $k + p > 1$ ,  $t + k \leq 1$ .

The quantity  $\epsilon$  is defined as to represent the smallest quantity available so that even the antonym of  $quantity(Q^{\text{Some}})$  is greater than  $quantity(Q^{\text{Almost all}})$ , i.e.,  $p < 1 - \epsilon$ .

**Definition 4 (Signum)** The signum of proposition  $\mathcal{P}$  is defined as

$$\text{signum}(\mathcal{P}) = \begin{cases} 1 & \text{if } \mathcal{P} \text{ is affirmative,} \\ 0 & \text{if } \mathcal{P} \text{ is negative.} \end{cases}$$

**Definition 5 (Distribution)** The distribution of term  $T$  in proposition  $\mathcal{P}$ ,  $\text{dist}(T, \mathcal{P})$ , equals to

1.  $quantity(Q)$  if  $\mathcal{P} = Q(T, X)$ ;
2.  $\epsilon$  if  $\mathcal{P} = Q(X, T)$  and  $\mathcal{P}$  is affirmative;
3.  $1$  if  $\mathcal{P} = Q(X, T)$  and  $\mathcal{P}$  is negative.

Below we introduce Extended Peterson's rules based on grades. †)

**Definition 6 (Peterson's rules)** Let  $\mathcal{S} = \langle \mathcal{P}_1, \mathcal{P}_2, \mathcal{C} \rangle$  be a syllogism such that  $S$  is the first formula of conclusion  $\mathcal{C}$ ,  $P$  is the second formula of conclusion  $\mathcal{C}$  and  $M$  is the middle formula.

#### 1. Rules of Distribution

- (R1)  $\text{dist}(M, \mathcal{P}_1) \otimes \text{dist}(M, \mathcal{P}_2) > 0$ ;  
 (R2a)  $\text{dist}(S, \mathcal{C}) \leq \text{dist}(S, \mathcal{P}_2)$ ;  
 (R2b)  $\text{dist}(P, \mathcal{C}) \leq \text{dist}(P, \mathcal{P}_1)$ ;

#### 2. Rules of Quality

- (R3)  $\text{signum}(\mathcal{P}_1) \vee \text{signum}(\mathcal{P}_2) = 1$ ;  
 (R4)  $\text{signum}(\mathcal{P}_1) \wedge \text{signum}(\mathcal{P}_2) = \text{signum}(\mathcal{C})$ .

## 3 Application

### PPx-III:

$\mathcal{P}_1$  : Almost all  $\underbrace{\text{shares of companies}}_{\text{middle formula}}$  grow with  $\underbrace{\text{growing economy}}_{\text{predicate}}$ .

$\mathcal{P}_2$  : Almost all  $\underbrace{\text{shares of companies}}_{\text{middle formula}}$  grow while  $\underbrace{\text{companies are on World Stock Markets}}_{\text{subject}}$ .

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$\mathcal{C}$  :  $Q_3$   $\underbrace{\text{companies being on World Stock Markets}}_{\text{subject}}$  have  $\underbrace{\text{growing economy}}_{\text{predicate}}$ .

†)Let us recall that syllogisms can have a prescription in four figures, where the position of the subject, predicate and middle formula depends

<b>Figure I</b>	<b>Figure II</b>	<b>Figure III</b>	<b>Figure IV</b>
$Q_1 M \text{ is } P$	$Q_1 P \text{ is } M$	$Q_1 M \text{ is } P$	$Q_1 P \text{ is } M$
$Q_2 S \text{ is } M$	$Q_2 S \text{ is } M$	$Q_2 M \text{ is } S$	$Q_2 M \text{ is } S$
$Q_3 S \text{ is } P$	$Q_3 S \text{ is } P$	$Q_3 S \text{ is } P$	$Q_3 S \text{ is } P$

Table 1: Forms of generalized syllogism with intermediate quantifiers

	R1	R2	R3	R4	R1,R2,R3,R4
<b>PPT</b>	FFTF	TTFE	TTTT	TTTT	FFFF
<b>PPF</b>	FFTF	TTFE	TTTT	TTTT	FFFF
<b>PPS</b>	FFTF	TTFE	TTTT	TTTT	FFFF
<b>PPI</b>	FFTF	TTTT	TTTT	TTTT	FFTF

- (GR1): This rule is trivially fulfilled for all conclusions, because  $dist(M, \mathcal{P}_1) = p > 0.5$  and  $dist(M, \mathcal{P}_2) > p > 0.5$ . Using the Definition 3  $dist(M, \mathcal{P}_1) \otimes dist(M, \mathcal{P}_2) > 0$ .
- (GR2a):  $dist(S, \mathcal{P}_2) = \epsilon$ .
- (GR2b):  $dist(P, \mathcal{P}_1) = \epsilon$ . For the valid syllogism, we have to find quantifier  $Q_3$  which has distribution at most  $\epsilon$  for the subject as well as for the predicate. It is fulfilled for “Some”.
- (GR3) and (GR4) are trivially fulfilled.

We can observe that the syllogism with the quantifier in both premises is valid with the quantifier “Some” in the conclusion only. We can observe that in the third figure, given the position of the subject and predicate in the antecedent, we can verify the validity of other non-trivial syllogisms with a particular conclusion in a similar way.

## 4 Discussion

From the Table 1 we can observe that for example the syllogism **PPT** fulfills the rule R1 in the third figure. (It is denoted by T.) In other figures the first rule is not fulfilled which is marked with the letter F. For a syllogism to be valid, it is necessary that all the rules in the given figure must be fulfilled. We can observe that only the syllogism **PPI** satisfies all the rules in the third figure, which we also proved in the analysis of the specific example above.

## References

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