Properties of Graded Peterson's Square of Opposition as Immediate Inferences

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Abstract

Immediate inferences are arguments where the conclusion is supported by just one premise. There are several ways to infer a conclusion from a premise. We can use conversion, obversion, contraposition or the properties of some structure of opposition. In this article, we will focus on the study of immediate inference for several forms of intermediate quantifiers that form a graded Peterson's square of opposition which describes properties of *contrary, contradictory, subcontrary, subaltern*, and *superaltern* between these quantifiers.

Keywords: Generalized Intermediate quantifiers; Graded Peterson's square of opposition; Immediate inference

1 Introduction

We use quantifiers to quantify the number of objects in everyday communication. In classical logic, we distinguish universal quantifier *All* and particular quantifier *Some.* Classical square of opposition has been explored in several publications including [1, 2, 3]. Square of the opposition contains classical quantifiers which we denote by capital letters **A,E,I,O** and describes properties of *contrary, contradictory, subcontrary, subaltern, superaltern* between these classical quantifiers. The square of oppositions is used to explain relationships between propositions within traditional formal logic but we can use it also for an immediate inference. **Immediate inference** is a type of inference where we infer another quantifier from one quantifier. The square of oppositions provides a structure for how different quantifiers can be related to each other and shows what inferential conclusions we can draw if one of the statements is true or false. There are several books that deal with this topic including [4, 5, 6].

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1.1 The main goal of this paper

The goal of this paper is to present immediate inferences in Fuzzy Natural Logic (FNL). For that, we will extend the classical approach in several ways. Firstly we will add intermediate quantifiers *Almost all, Most, Many* which were introduced by Peterson [7] and their mathematical definition was introduced by Novák [8] in Łukasiewicz fuzzy type theory (Ł-FTT). Instead of the square of opposition, we will use its generalized form so-called graded Peterson's square of oppositions [9]. The truth values will be from the interval $[0, 1]$ since we use Ł-FTT. Let us note that we will deal with properties of contrary and subaltern in this paper due to the page limitation but presented ideas will work similarly for contradictory, subcontrary and superaltern.

2 Preliminaries

2.1 Mathematical bacground

Intermediate quantifiers are formally defined within a dedicated theoretical framework called T^{IQ}, which is a specialized component of Łukasiewicz Fuzzy Type Theory (Ł-FTT). In this context, we operate under the assumption that the underlying algebra of truth values is an MV_Δ-algebra, denoted as $\mathscr{E} = \langle E, \vee, \wedge, \otimes, \rightarrow, \mathbf{0}, \mathbf{1}, \Delta\rangle$ where $\mathbf{0}$ where and $\mathbf{1}$ represent the minimal and maximal elements, respectively. In most cases, we work with the standard Łukasiewicz MV_Δ-algebra, where the domain of truth degrees is given by the interval $E = [0, 1]$.

2.2 Mathematical definition of intermediate quantifiers

Below we recall mathematical definitions of fuzzy intermediate quantifiers that define graded Peterson's square of opposition. For the detail see [10].

Definition 1 *Let Ev be a formula representing an evaluative expression*[†)]*, x be variables and A, B, z be formulas. Then either of the formulas*

$$(Q^{\forall}_{Ev}\, x)(B, A) \equiv (\exists z)[(\forall x)((B|z)\, x \Rightarrow Ax) \wedge Ev((\mu(B))(B|z))], \tag{1}$$

$$(Q^{\exists}_{Ev}\, x)(B, A) \equiv (\exists z)[(\exists x)((B|z)x \wedge Ax) \wedge Ev((\mu(B))(B|z))]. \tag{2}$$

construes the sentence "⟨Quantifier⟩ B's are A".

Since we are limited by the number of pages, we cannot give all the examples of intermediate quantifiers in detail. Let us note that the specific examples of quantifiers are based on evaluative expressions. For example, the quantifier "Almost all" is represented by the evaluative expression "extremely big" and is marked with the letter **P**, for predominant. For further markings, we refer the reader to previous publications.

3 Properties of contrary and subaltern between quantifiers

In this section, we will focus on the study of immediate inferences which are represented by properties contrary and subaltern, which define graded Peterson's square of opposition.

[†)] The semantics of evaluative linguistic expressions is interpreted in a special formal theory T^{Ev} of Ł-FTT. (see [11]).

Proposition 2 (contrary) *Let P_1, P_2 be quantifiers such that P_1 and P_2 are contraries in the model $\mathscr{M}$. Then holds true that*

$$\mathscr{M}(P_2) \leq 1 - \mathscr{M}(P_1).$$

Proposition 3 (subaltern) *Let P_1, P_2 be quantifiers such that P_2 is subaltern of P_1 in the model $\mathscr{M}$. Then holds true that*

$$\mathscr{M}(P_1) \leq \mathscr{M}(P_2).$$

3.0.1 Demonstrative example

The idea is to use information about the truth value of the quantifier and information about the relationship of this known quantifier with other quantifiers. An example that will be dedicated to natural language claims related to **disinformation**. The most vulnerable group are seniors over 65. Below is a typical example.

- **Positive:** Almost all seniors realize that false news spreads quickly.

Below we introduce the examples of intermediate quantifiers in the particular model. Let there be a model $\mathscr{M} \models T$. Let us know that
P: "Almost all seniors realize that false news spreads quickly." *has truth value 0.9.*
Therefore, we know that **E:** "No seniors realize that false news spreads quickly" is contrary with **P**.
From that we can infer "No seniors realize that false news spreads quickly." *has truth value* $[0, 0.1]$ using Proposition 2 as follows $\mathscr{M}(\mathbf{E}) \leq 1 - 0.9$, $\mathscr{M}(\mathbf{E}) \leq 0.1$

Similarly, we can infer truth values of other quantifiers using Propositions 2-3 and using relationship of other quantifiers with a quantifier **P**. Below we introduce several examples:

- "**All** seniors realize that false news spreads quickly." *has truth value* $[0, 0.9]$. (subaltern)
- "**Most, Many, Some** seniors realize that false news spreads quickly."*all these has truth value* $[0.9, 1]$. (subaltern)
- "**No** seniors realize that false news spreads quickly.","**Almost all, Most, Many** seniors do not realize that false news spreads quickly." *all these has truth value* $[0, 0.1]$. (contrary)

We can see that for some quantifiers we obtained quite small interval of possible truth values. On the other hand for some quantifiers we obtained very wide intervals of possible truth values which do not give as much information. We are not able to directly infer truth value of quantifier **O**: Some seniors do not realize that false news spreads quickly.

4 Discussion and Future work

We showed how to use contrary and subaltern as immediate inferences in fuzzy natural logic. In general, for this inference it is necessary to know the truth of the quantifier and the relation of that known quantifier with the others. To describe the relations we have used graded Peterson's square of oppositions however there are other structures of oppositions e.g. graded hexagon of opposition [12, 13] or graded cubes of opposition [14].

In further scientific research, we will build on this publication and focus on other properties of the inference that it belongs to conversion, obversion, and contraposition.

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